

limiting high temperature value of the γ s more slowly than γ . Of course this theorem leads one further to anticipate that γ'' for copper and aluminum should not be independent of temperature in the entire range in which this holds for γ . For the alkali halides for which BARRON *et al.*⁽²¹⁾ have proved the accuracy of the quasi-harmonic approximation to the thermal thermodynamic functions at moderate temperatures, one would again expect that γ at atmospheric pressure should not vary significantly with temperature in a region around and below the pertinent Θ_2 . The measurements of RUBIN *et al.*⁽²⁵⁾ prove that this is true for sodium chloride.

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APPENDIX

The Quasi-Harmonic Approximation at Moderate Temperatures and the Debye Model

At temperatures above $h\nu_m/2\pi k$, where ν_m is the highest vibrational frequency of the solid, the thermodynamic functions of a quasi-harmonic non-metal are represented by their Thirring-Stern expansions⁽¹⁰⁾ in inverse powers of the absolute temperature:

$$\frac{F_{vib}}{3NkT} = \frac{F_{th}}{3NkT} + \frac{1}{2} \frac{h}{kT} \mu_1 = \ln \left[\frac{h}{kT} (\prod_j \nu_j)^{1/3N} \right] \quad (A.1)$$

$$- \sum_{n=1}^{\infty} (-1)^n \frac{B_{2n}}{2n(2n)!} \left(\frac{h}{kT} \right)^{2n} \mu_{2n}$$

$$\frac{S}{3Nk} = -\ln \left[\frac{h}{kT} (\prod_j \nu_j)^{1/3N} \right]$$

$$+ 1 - \sum_{n=1}^{\infty} (-1)^n \frac{B_{2n}}{(2n)!} \frac{2n-1}{2n} \left(\frac{h}{kT} \right)^{2n} \mu_{2n} \quad (A.2)$$

$$\frac{W_{vib.}}{3NkT} = \frac{W_{th.}}{3NkT} + \frac{1}{2} \frac{h}{kT} \mu_1 = 1$$

$$- \sum_{n=1}^{\infty} (-1)^n \frac{B_{2n}}{(2n)!} \left(\frac{h}{kT} \right)^{2n} \mu_{2n} \quad (A.3)$$

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